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LARGE-SCALE MAGNETIC FIELD ANALYSES

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1 INTRODUCTION

For the last several years, we have been considering mixed finite element approximations for three-dimensional eddy current problems, using the Nedelec element of simplex type and the conventional piecewise linear tetrahedral element. These formulations are extensions of some mixed ones proposed for magnetostatic and electrostatic problems; see [Kikuchi, 1989a] and [Kikuchi, 1989b]. Up to the present, we have shown the effectiveness of these formulations; see [Kanayama et al., 1997] and [Kanayama and Kikuchi, 1999]. These computations using a direct method to solve the resultant linear system have consumed huge memories, disk spaces, and CPU time. This implies that an iterative solver may be required for larger scale analyses; particularly for our present goal to solve magnetic field problems with about one million DOF.

In our mixed formulation introducing the Lagrange multipliers, the resultant linear system is asymmetric and indefinite, which causes difficulties in choosing the iterative solver. On the other hand, it is easy to find that these Lagrange multipliers turn out to be zero. Therefore it is quite desirable to eliminate these extra quantities. So we introduce a new formulation based on an initial step of the iterative scheme for magnetostatic problems in [Kikuchi and Fukuhara, 1995]. Owing to this formulation, we show that some iterative solvers are appropriate for magnetic field analyses.

This paper is organized as follows. In Section 2 we give a conventional formulation of three-dimensional eddy current problems, which are known as the H-J formulation, and propose a new one where the Lagrange multipliers are eliminated. In Section 3 we introduce two models, and show their numerical results. Finally concluding remarks are given in Section 4.

2 FORMULATIONS

2.1 The *H*-*J* formulation

Let Ω be a domain consisting of a conducting region R and a non-conducting region S. Here assume that Ω, R and S are polyhedorons. Let us denote by $\partial\Omega$ the boundary of Ω, and by Γ the interface between R and S. Let $(0,T)$ be a time interval. Using the magnetic field $H\,[\mathrm{A/m}]$ and the eddy current $Je\,[\mathrm{A/m^2}]$ as unknown functions, we consider a three-dimensional eddy current problem:

$$\begin{cases} \mathrm{rot}\, H = Jo + Je & \text{in } R\times(0,T), \quad (1a)\\ \mathrm{rot}\, H = Jo & \text{in } S\times(0,T), \quad (1b)\\ \mathrm{div}(\mu H) = 0 & \text{in } \Omega\times(0,T), \quad (1c)\\ \mathrm{rot}(\sigma^{-1}Je) = -\partial_t(\mu H) & \text{in } R\times(0,T), \quad (1d)\\ \mathrm{div}\, Je = 0 & \text{in } R\times(0,T), \quad (1e)\end{cases}$$

where Jo is an excitation current $[\mathrm{A/m^2}]$, μ is the permeability $[\mathrm{H/m}]$, and σ is the conductivity $[\mathrm{S/m}]$. Throughout this paper, for simplicity, let us assume that μ is a piecewise positive constant, and that σ is a positive constant.

On the interface Γ, the following conditions are imposed:

$$\begin{cases} \big[(\mu H)\cdot n\big]_R^S = 0, & (2a)\\ \big[H\times n\big]_R^S = 0, & (2b)\\ Je\cdot n = 0, & (2c)\end{cases}$$

where $[\cdot]_R^S$ denotes the difference between traces on Γ from S to R, and n denotes a unit normal from S to R. On the boundary $\partial\Omega$, the following condition is imposed:

$$(\mu H)\cdot n = 0, \tag{3}$$

where n denotes a unit normal from Ω to the boundary. Adding appropriate initial conditions, the description of the problem is completed.

Let i be the imaginary unit, and ω the angular frequency [rad/s]. Replacing the time derivative ∂_t by $-i\omega$, we consider the sinusoidal field. As usual, considering that all variables have complex values (for example $H = H_r + iH_i$), we have the following problem:

$$\begin{cases} \mathrm{rot}\, H_r = Jo_r + Je_r, & \mathrm{rot}\, H_i = Jo_i + Je_i & \text{in } R, \quad (4a)\\ \mathrm{rot}\, H_r = Jo_r, & \mathrm{rot}\, H_i = Jo_i & \text{in } S, \quad (4b)\\ \mathrm{div}(\mu H_r) = 0, & \mathrm{div}(\mu H_i) = 0 & \text{in } \Omega, \quad (4c)\\ \mathrm{rot}(\sigma^{-1}Je_r) = -\omega\mu H_i, & \mathrm{rot}(\sigma^{-1}Je_i) = \omega\mu H_r & \text{in } R, \quad (4d)\\ \mathrm{div}\, Je_r = 0, & \mathrm{div}\, Je_i = 0 & \text{in } R \quad (4e)\end{cases}$$

with the boundary conditions:

$$
\begin{cases}
\big[(\mu H_r)\cdot n\big]_R^S = 0, & \big[(\mu H_i)\cdot n\big]_R^S = 0 \quad \text{on } \Gamma, & (4f)\\
\big[H_r \times n\big]_R^S = 0, & \big[H_i \times n\big]_R^S = 0 \quad \text{on } \Gamma, & (4g)\\
Je_r \cdot n = 0, & Je_i \cdot n = 0 \quad \text{on } \Gamma, & (4h)\\
(\mu H_r)\cdot n = 0, & (\mu H_i)\cdot n = 0 \quad \text{on } \partial\Omega, & (4i)
\end{cases}
$$

where each subscript r (or i) denotes the real (or imaginary) part of each variable.

As usual, let $L^2(\Omega)$ be the space of functions defined in Ω and square summable in Ω, and $H^1(\Omega)$ the space of functions in $L^2(\Omega)$ with derivatives up to the first order. Let us denote by $(\,\cdot\,,\,\cdot\,)_\Omega$ and by $(\,\cdot\,,\,\cdot\,)_R$ the inner product of $L^2(\Omega)$ and $L^2(R)$, respectively. Recall some function spaces introduced in [Kikuchi and Fukuhara, 1995]:

$$
\begin{gathered}
L^2(\Omega,\mu) \equiv L^2(\Omega) \text{ equipped with the norm } \|v\|_\mu \equiv \|\mu^{1/2} v\|_{L^2(\Omega)},\\
H(\mathrm{rot};\Omega,\mu) \equiv \big\{ v \in L^2(\Omega,\mu)^3;\ \mathrm{rot}\, v \in L^2(\Omega)^3 \big\},\\
H(\mathrm{rot};\Omega) \equiv H(\mathrm{rot};\Omega,1).
\end{gathered}
$$

The magnetic field, the eddy current, and the Lagrange multipliers are sought from the following spaces:

$$
T = H(\mathrm{rot};R), \quad U = H^1(R), \quad V = H(\mathrm{rot};\Omega,\mu), \quad W = H^1(\Omega).
$$

We consider the weak formulation of (4): given $Jo_r,\ Jo_i \in L^2(\Omega)^3$, find $Je_r,\ Je_i \in T$, $q_r,\ q_i \in U$, $H_r,\ H_i \in V$, and $p_r,\ p_i \in W$ such that

$$
\begin{cases}
(\mathrm{rot}\, H_r, \mathrm{rot}\, H_r^*)_\Omega + (\mathrm{grad}\, p_r, \mu H_r^*)_\Omega & \\
\qquad = (Jo_r, \mathrm{rot}\, H_r^*)_\Omega + (Je_r, \mathrm{rot}\, H_r^*)_R & \text{for } H_r^* \in V, \quad (5a)\\
(\mathrm{rot}\, H_i, \mathrm{rot}\, H_i^*)_\Omega + (\mathrm{grad}\, p_i, \mu H_i^*)_\Omega & \\
\qquad = (Jo_i, \mathrm{rot}\, H_i^*)_\Omega + (Je_i, \mathrm{rot}\, H_i^*)_R & \text{for } H_i^* \in V, \quad (5b)\\
(\mu H_r, \mathrm{grad}\, p_r^*)_\Omega = 0 & \text{for } p_r^* \in W, \quad (5c)\\
(\mu H_i, \mathrm{grad}\, p_i^*)_\Omega = 0 & \text{for } p_i^* \in W, \quad (5d)\\
(\mathrm{rot}(\sigma^{-1} Je_r), \mathrm{rot}\, Je_r^*)_R + (\mathrm{grad}\, q_r, Je_r^*)_R & \\
\qquad = -(\omega\mu H_i, \mathrm{rot}\, Je_r^*)_R & \text{for } Je_r^* \in T, \quad (5e)\\
(\mathrm{rot}(\sigma^{-1} Je_i), \mathrm{rot}\, Je_i^*)_R + (\mathrm{grad}\, q_i, Je_i^*)_R & \\
\qquad = (\omega\mu H_r, \mathrm{rot}\, Je_i^*)_R & \text{for } Je_i^* \in T, \quad (5f)\\
(Je_r, \mathrm{grad}\, q_r^*)_R = 0 & \text{for } q_r^* \in U, \quad (5g)\\
(Je_i, \mathrm{grad}\, q_i^*)_R = 0 & \text{for } q_i^* \in U. \quad (5h)
\end{cases}
$$

Although they are determined up to an additive constant, the Lagrange multipliers may vanish, which is proved easily by setting $H_r^* = \mathrm{grad}\, p_r$, $H_i^* = \mathrm{grad}\, p_i$, $Je_r^* = \mathrm{grad}\, q_r$, and $Je_i^* = \mathrm{grad}\, q_i$.

We consider the decomposition of Ω into tetrahedrons, and approximate the magnetic field and the eddy current by the Nedelec element of simplex type and the Lagrange multipliers by the conventional piecewise linear element. We consider the mixed finite element approximation of (5): find Je_{rh}, $Je_{ih} \in T_h$, q_{rh}, $q_{ih} \in U_h$, H_{rh}, $H_{ih} \in V_h$, and p_{rh}, $p_{ih} \in W_h$ such that

$$\left\{\begin{array}{lll}
(\mathrm{rot}\, H_{rh}, \mathrm{rot}\, H^*_{rh})_\Omega + (\mathrm{grad}\, p_{rh}, \mu H^*_{rh})_\Omega & & \\
\qquad\qquad = (Jo_r, \mathrm{rot}\, H^*_{rh})_\Omega + (Je_{rh}, \mathrm{rot}\, H^*_{rh})_R & \text{for } H^*_{rh} \in V_h, & \text{(6a)} \\
(\mathrm{rot}\, H_{ih}, \mathrm{rot}\, H^*_{ih})_\Omega + (\mathrm{grad}\, p_{ih}, \mu H^*_{ih})_\Omega & & \\
\qquad\qquad = (Jo_i, \mathrm{rot}\, H^*_{ih})_\Omega + (Je_{ih}, \mathrm{rot}\, H^*_{ih})_R & \text{for } H^*_{ih} \in V_h, & \text{(6b)} \\
(\mu H_{rh}, \mathrm{grad}\, p^*_{rh})_\Omega = 0 & \text{for } p^*_{rh} \in W_h, & \text{(6c)} \\
(\mu H_{ih}, \mathrm{grad}\, p^*_{ih})_\Omega = 0 & \text{for } p^*_{ih} \in W_h, & \text{(6d)} \\
(\mathrm{rot}(\sigma^{-1} Je_{rh}), \mathrm{rot}\, Je^*_{rh})_R + (\mathrm{grad}\, q_{rh}, Je^*_{rh})_R & & \\
\qquad\qquad = -(\omega\mu H_{ih}, \mathrm{rot}\, Je^*_{rh})_R & \text{for } Je^*_{rh} \in T_h, & \text{(6e)} \\
(\mathrm{rot}(\sigma^{-1} Je_{ih}), \mathrm{rot}\, Je^*_{ih})_R + (\mathrm{grad}\, q_{ih}, Je^*_{ih})_R & & \\
\qquad\qquad = (\omega\mu H_{rh}, \mathrm{rot}\, Je^*_{ih})_R & \text{for } Je^*_{ih} \in T_h, & \text{(6f)} \\
(Je_{rh}, \mathrm{grad}\, q^*_{rh})_R = 0 & \text{for } q^*_{rh} \in U_h, & \text{(6g)} \\
(Je_{ih}, \mathrm{grad}\, q^*_{ih})_R = 0 & \text{for } q^*_{ih} \in U_h, & \text{(6h)}
\end{array}\right.$$

where subscript h denotes approximate functions and spaces. As in the continuous case, the Lagrange multipliers vanish again.

2.2 ITERATION METHOD

Let τ be a positive constant. For the weak formulation (5), let us consider a perturbation problem: find Je_r, $Je_i \in T$, q_r, $q_i \in U$, H_r, $H_i \in V$, and p_r, $p_i \in W$ such that

$$\left\{\begin{array}{lll}
(\mathrm{rot}\, H_r, \mathrm{rot}\, H^*_r)_\Omega + (\mathrm{grad}\, p_r, \mu H^*_r)_\Omega + \tau(\mu H_r, H^*_r)_\Omega & & \\
\qquad\qquad = (Jo_r, \mathrm{rot}\, H^*_r)_\Omega + (Je_r, \mathrm{rot}\, H^*_r)_R & \text{for } H^*_r \in V, & \text{(7a)} \\
(\mathrm{rot}\, H_i, \mathrm{rot}\, H^*_i)_\Omega + (\mathrm{grad}\, p_i, \mu H^*_i)_\Omega + \tau(\mu H_i, H^*_i)_\Omega & & \\
\qquad\qquad = (Jo_i, \mathrm{rot}\, H^*_i)_\Omega + (Je_i, \mathrm{rot}\, H^*_i)_R & \text{for } H^*_i \in V, & \text{(7b)} \\
(\mu H_r, \mathrm{grad}\, p^*_r)_\Omega = 0 & \text{for } p^*_r \in W, & \text{(7c)} \\
(\mu H_i, \mathrm{grad}\, p^*_i)_\Omega = 0 & \text{for } p^*_i \in W, & \text{(7d)} \\
(\mathrm{rot}(\sigma^{-1} Je_r), \mathrm{rot}\, Je^*_r)_R + (\mathrm{grad}\, q_r, Je^*_r)_R + \tau(Je_r, Je^*_r)_R & & \\
\qquad\qquad = -(\omega\mu H_i, \mathrm{rot}\, Je^*_r)_R & \text{for } Je^*_r \in T, & \text{(7e)} \\
(\mathrm{rot}(\sigma^{-1} Je_i), \mathrm{rot}\, Je^*_i)_R + (\mathrm{grad}\, q_i, Je^*_i)_R + \tau(Je_i, Je^*_i)_R & & \\
\qquad\qquad = (\omega\mu H_r, \mathrm{rot}\, Je^*_i)_R & \text{for } Je^*_i \in T, & \text{(7f)} \\
(Je_r, \mathrm{grad}\, q^*_r)_R = 0 & \text{for } q^*_r \in U, & \text{(7g)} \\
(Je_i, \mathrm{grad}\, q^*_i)_R = 0 & \text{for } q^*_i \in U. & \text{(7h)}
\end{array}\right.$$

As in Subsection 2.1, the Lagrange multipliers vanish again. So we obtain the following problem that does not include the Lagrange multipliers: find H_r, $H_i \in V$ and Je_r, $Je_i \in T$ such that

$$\begin{cases} (\mathrm{rot}\, H_r, \mathrm{rot}\, H_r^*)_\Omega + \tau(\mu H_r, H_r^*)_\Omega \\ \qquad = (Jo_r, \mathrm{rot}\, H_r^*)_\Omega + (Je_r, \mathrm{rot}\, H_r^*)_R \quad \text{for } H_r^* \in V, & \text{(8a)} \\ (\mathrm{rot}\, H_i, \mathrm{rot}\, H_i^*)_\Omega + \tau(\mu H_i, H_i^*)_\Omega \\ \qquad = (Jo_i, \mathrm{rot}\, H_i^*)_\Omega + (Je_i, \mathrm{rot}\, H_i^*)_R \quad \text{for } H_i^* \in V, & \text{(8b)} \\ (\mathrm{rot}(\sigma^{-1} Je_r), \mathrm{rot}\, Je_r^*)_R + \tau(Je_r, Je_r^*)_R \\ \qquad = -(\omega\mu H_i, \mathrm{rot}\, Je_r^*)_R \quad \text{for } Je_r^* \in T, & \text{(8c)} \\ (\mathrm{rot}(\sigma^{-1} Je_i), \mathrm{rot}\, Je_i^*)_R + \tau(Je_i, Je_i^*)_R \\ \qquad = (\omega\mu H_r, \mathrm{rot}\, Je_i^*)_R \quad \text{for } Je_i^* \in T. & \text{(8d)} \end{cases}$$

Using the Nedelec element of simplex type, we propose an approximation problem that does not include the Lagrange multipliers: find H_{rh}, $H_{ih} \in V_h$ and Je_{rh}, $Je_{ih} \in T_h$ such that

$$\begin{cases} (\mathrm{rot}\, H_{rh}, \mathrm{rot}\, H_{rh}^*)_\Omega + \tau(\mu H_{rh}, H_{rh}^*)_\Omega \\ \qquad = (Jo_r, \mathrm{rot}\, H_{rh}^*)_\Omega + (Je_{rh}, \mathrm{rot}\, H_{rh}^*)_R \quad \text{for } H_{rh}^* \in V_h, & \text{(9a)} \\ (\mathrm{rot}\, H_{ih}, \mathrm{rot}\, H_{ih}^*)_\Omega + \tau(\mu H_{ih}, H_{ih}^*)_\Omega \\ \qquad = (Jo_i, \mathrm{rot}\, H_{ih}^*)_\Omega + (Je_{ih}, \mathrm{rot}\, H_{ih}^*)_R \quad \text{for } H_{ih}^* \in V_h, & \text{(9b)} \\ (\mathrm{rot}(\sigma^{-1} Je_{rh}), \mathrm{rot}\, Je_{rh}^*)_R + \tau(Je_{rh}, Je_{rh}^*)_R \\ \qquad = -(\omega\mu H_{ih}, \mathrm{rot}\, Je_{rh}^*)_R \quad \text{for } Je_{rh}^* \in T_h, & \text{(9c)} \\ (\mathrm{rot}(\sigma^{-1} Je_{ih}), \mathrm{rot}\, Je_{ih}^*)_R + \tau(Je_{ih}, Je_{ih}^*)_R \\ \qquad = (\omega\mu H_{rh}, \mathrm{rot}\, Je_{ih}^*)_R \quad \text{for } Je_{ih}^* \in T_h. & \text{(9d)} \end{cases}$$

The original algorithm in [Kikuchi and Fukuhara, 1995] goes on to an iterative step using the solution of (9) as initial functions. From practical point of view, an appropriate choice of τ implies that the solution of (9) is correct enough. Therefore, in this paper, we consider only the "initial step" (9).

3 COMPUTATIONAL MODEL

3.1 A cake model

Let us consider an infinite solenoidal coil including a conductor with radius 0.1 [m]; see Figure 1. By its symmetry, it suffices to consider only a sectoral domain as in Figure 2; so this model is called a "cake" model. This model is a three-dimensional extension of one described in [Nakata and Takahashi, 1986]. The permeability μ is $4\pi \times 10^{-7}$ [H/m], the conductivity σ is 7.7×10^6 [S/m], the frequency f is 60 [Hz] ($\omega = 2\pi f$), and the absolute value of the real (or imaginary) part of the excitation current $|Jo_r|$ (or $|Jo_i|$) is 50 (or 0) [A/m^2]. The numerical parameter τ is 1.0×10^{-4}.

The model is considered only in a sectoral domain whose central angle is 20°, and the domain is decomposed into tetrahedrons; see Figure 2. The number of nodal points, elements and DOF are 695, 324 and 1772, respectively. Let Γ_1 be the cross-section including the z-axis, and Γ_2 a union of the upper plane, the bottom plane and the surface of the cylinder. The following boundary conditions are imposed on $\partial\Omega$, though these conditions are slightly different from those in (3):

$$\begin{cases} (\mu H) \cdot n = 0 \ \text{ on } \ \Gamma_1 \cap \partial\Omega, \qquad H \times n = 0 \ \text{ on } \ \Gamma_2 \cap \partial\Omega, & \text{(10a)} \\ Je \times n = 0 \ \text{ on } \ \Gamma_1 \cap \partial R, \qquad Je \cdot n = 0 \ \text{ on } \ \Gamma_2 \cap \partial R, & \text{(10b)} \end{cases}$$

where ∂R is the boundary of R. Boundary conditions on Γ are described in (2). Computation was performed on Sun UltraSPARC 200MHz with 1 CPU by using the PETSc library (see [Balay et al., 1998]) as for solvers.

At first we compare the present results by the formulation (9) with the previous ones by the conventional H-J formulation in [Kanayama et al., 1998]. In the present results, LU decomposition is used. Figure 3 shows the z component of the approximate magnetic field H_{rh} in the conductor versus the radius r along the line with $\theta = 10°$ and $z = 0.05$m. A solid line denotes the present results, and the broken line denotes the previous ones. The present results almost agree with the previous ones.

Next we investigate the reliable iterative solver for the resultant linear system. Unfortunately, as for the conventional H-J formulation, there is no reliable iterative solver which is still effective for larger problems. Here two iterative solvers are chosen; one is a restart version of the Generalized Minimum Residual method (GMRES(m)) where the restart value of the iteration m is 100, 1772; the other is BiConjugate Gradient Stabilized method (Bi-CGSTAB). As for the preconditioner, the incomplete LU factorization of level zero ($ILU(0)$) and the Jacobi preconditioner are used. Zero vector was chosen as the initial vector of each iterative solver. Each process was stopped as soon as the residual norm $\|M^{-1}(b - Ax)\|/\|M^{-1}b\|$ with the preconditioner M was reduced by a factor of $\varepsilon = 10^{-7}$. Figure 4 shows the profiles of the residual norm $\|M^{-1}(b - Ax)\|/\|M^{-1}b\|$ versus the number of iterations. With each iterative solver, iteration for the resultant linear system converges except for GMRES(100) with the Jacobi preconditioner. Figure 4 also shows that GMRES(m) is better than Bi-CGSTAB.

3.2 TEAM Problem 7

Let us consider a benchmark problem, Problem 7, given by the Testing Electromagnetic Analysis Methods (TEAM) Workshop. Problem 7 is a three-dimensional eddy current problem of asymmetrical conductor with a hole; see Figure 5. The permeability μ is $4\pi \times 10^{-7}$ [H/m], the conductivity σ is 3.526×10^7 [S/m], the frequency f is 50 [Hz] ($\omega = 2\pi f$), and the absolute value of the real (or imaginary) part of the excitation current $|Jo_r|$ (or $|Jo_i|$) is 1.0968×10^6 (or 0) [A/m^2]. The numerical parameter τ is 1.0×10^{-4}.

As in Figure 6, the domain Ω is decomposed into tetrahedrons. The number of nodal points, elements and DOF are 85833, 60000 and 170133, respectively. Since the domain

Ω is large enough, we assume that the normal component of the magnetic flux density vanishes on the whole boundary $\partial\Omega$. Boundary conditions on Γ are described in (2). Computation of the TEAM model was performed on an Alpha cluster with 10 CPUs in the ADVENTURE project. GMRES(2000) with the Jacobi preconditioner in the PETSc library is used to solve the resultant linear system. From the results in the cake model, $ILU(0)$ may be better than Jacobi. However, the Jacobi preconditioner is used here, because the $ILU(0)$ preconditioner goes wrong in the parallel computing on the Alpha cluster now. Zero vector was chosen as the initial vector of GMRES(2000).

Figure 7 shows the profiles of the residual norm $\|M^{-1}(b - Ax)\| / \|M^{-1}b\|$ versus the number of iterations. The residual becomes smaller until the restart number 2000.

4 CONCLUDING REMARKS

We have introduced a new scheme for three-dimensional eddy current problems based on the initial step of an iteration scheme for magnetostatic problems.

At first we have compared the present results by the new scheme with the previous ones by the conventional H-J formulation, and have shown that the present results almost agree with the previous ones. Next we have investigated reliable iterative solvers of the resultant linear system. Thanks to the formulation where the Lagrange multipliers are eliminated, we have avoided difficulties with respect to the indefiniteness of matrices. Therefore, we have shown that, by GMRES(m) and Bi-CGSTAB (with both ILU(0) and Jacobi preconditioners), iteration for the resultant linear system converges. Finally we have considered Problem 7 in TEAM Workshop. Using GMRES(2000) with the Jacobi preconditioner, we have solved the resultant linear system.

As stated above, we have been investigating new formulations where the Lagrange multipliers are eliminated. In the case of magnetostatic problems, Conjugate Gradient method (CG) can be used to solve the resultant linear system, which implies the possibility of larger scale analyses. So we are trying problems with hundreds thousands DOF. Although scale of this computation is not small, it is not enough for our present goal. Therefore we are also trying to apply domain decomposition method to magnetic field analyses.

REFARENCES

Balay, S., Gropp, W., Mclnnes, L. C., and Smith, B. (1998). *PETSc 2.0 Users Manual.* Argonne National Laboratory, `http://www.mcs.anl.gov/petsc`. ANL-95/11-Revision 2.0.24.

Kanayama, H., Ikeguchi, S., and Kikuchi, F. (1997). 3-D eddy current analysis using the Nedelec elements. In *Proceedings of ICES '97*, pages 277–282.

Kanayama, H. and Kikuchi, F. (1999). 3-D eddy current computation using the Nedelec elements. *Information*, 2(1):37–46.

Kanayama, H., Shioya, R., Nakiri, K., Saito, M., and Ogino, M. (1998). An application to magnetic field analysis of iterative solvers for linear equations with non-symmetric matrices. In *Proceedings of JSST.* (in Japanese).

Kikuchi, F. (1989a). Mixed formulations for finite element analysis of magnetostatic and electrostatic problems. *Japan J. Appl. Math.*, 6:209–221.

Kikuchi, F. (1989b). On a discrete compactness property for the Nedelec finite elements. *J. Fac. Sci. Univ. Tokyo, Sect. IA Math.*, 36:479–490.

Kikuchi, F. and Fukuhara, M. (1995). An iteration method for finite element analysis of magnetostatic problems. *Lecture Notes in Num. Appl. Anal.*, 14:93–105.

Nakata, T. and Takahashi, N. (1986). *Finite Element Methods in Electrical Engineering.* Morikita Shuppan, 2nd edition. (in Japanese).

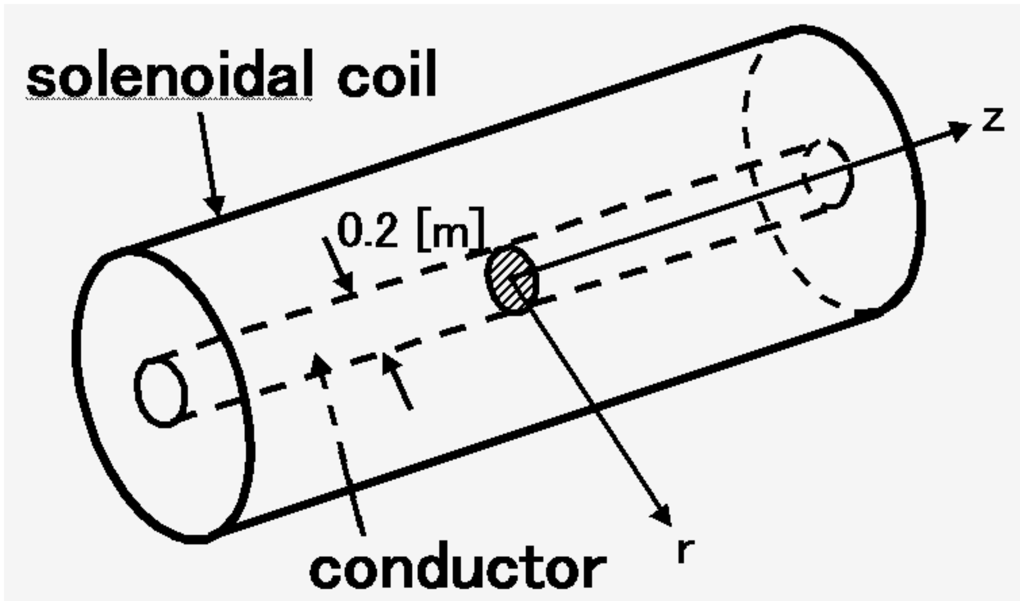

Figure 1: An infinite solenoidal coil.

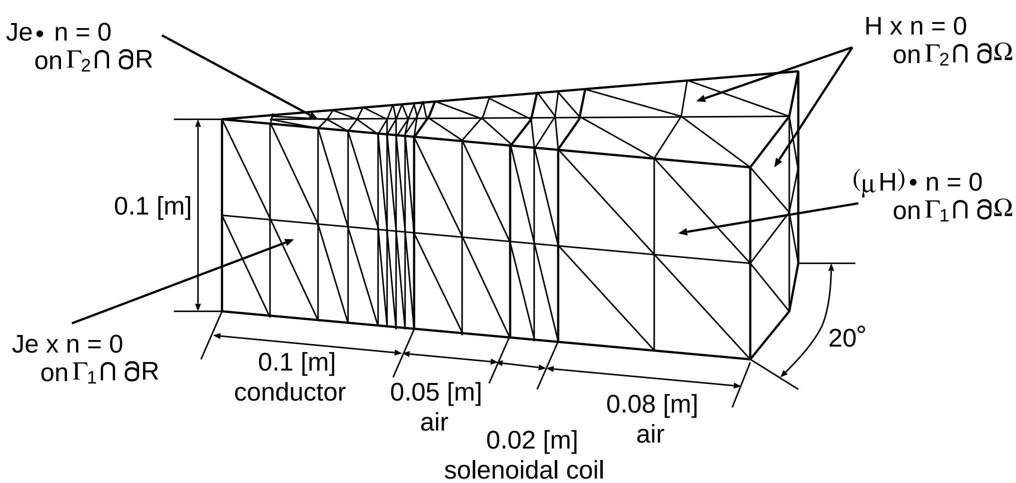

Figure 2: A finite element mesh for the "cake" model.

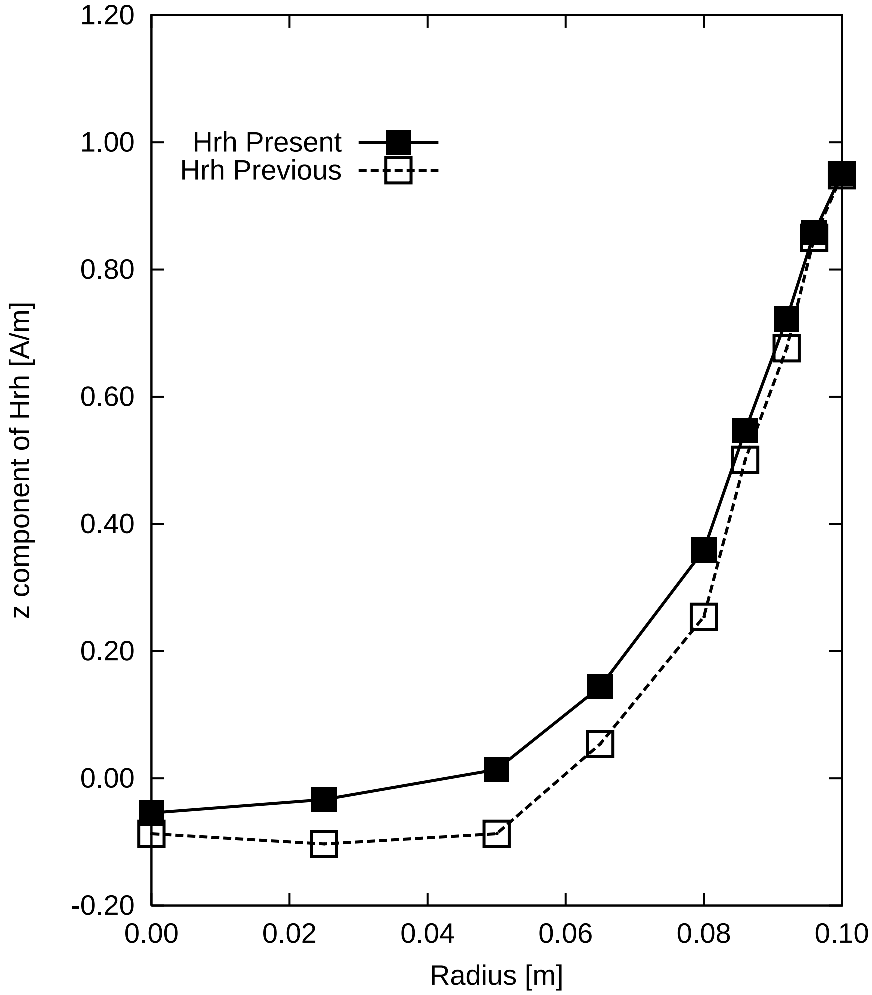

Figure 3: The z component of the approximate magnetic field H_{rh} in the conductor.

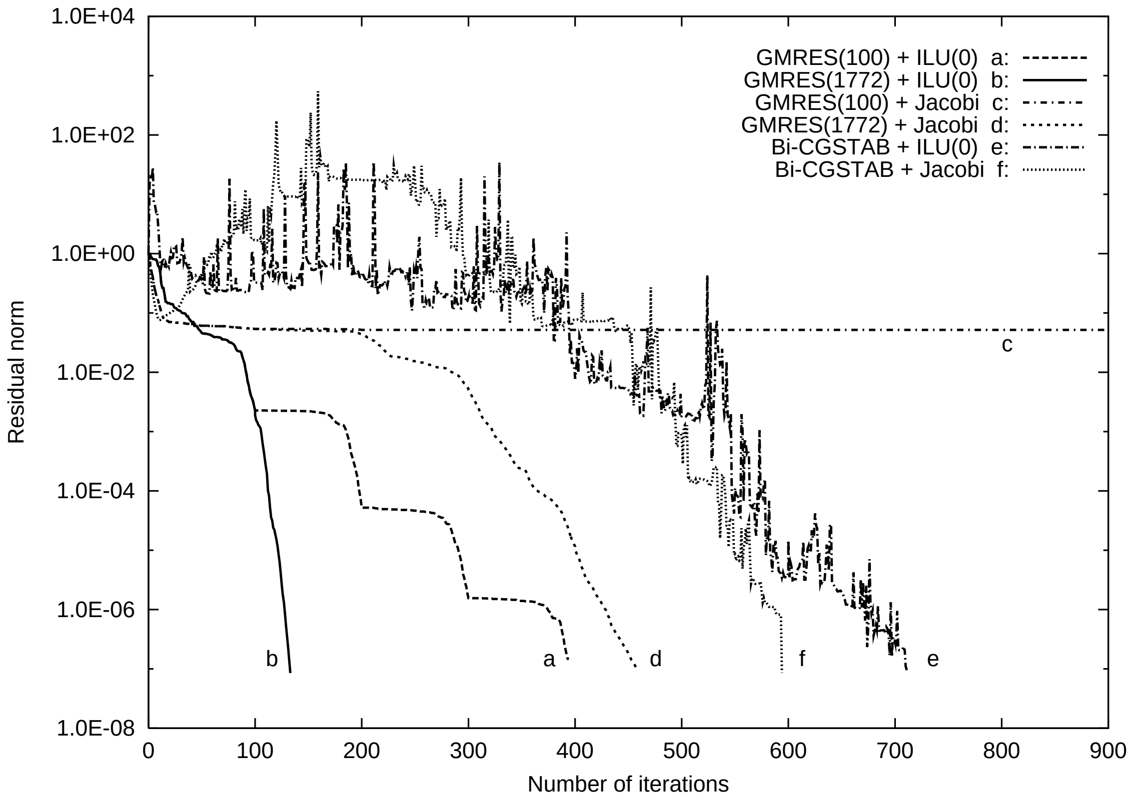

Figure 4: The profile of the residual norm $\|M^{-1}(b - Ax)\|/\|M^{-1}b\|$ versus the number of iterations.

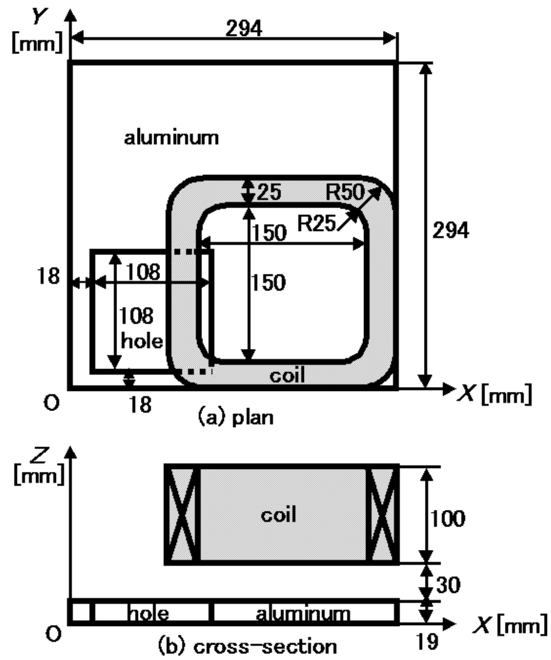

Figure 5: Asymmetrical conductor with a hole in TEAM Problem 7.

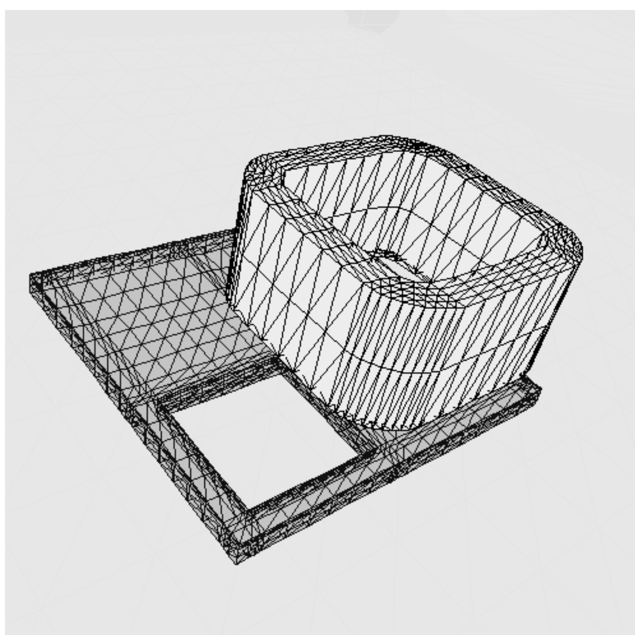

Figure 6: A finite element mesh around the coil and the conductor for the TEAM model.

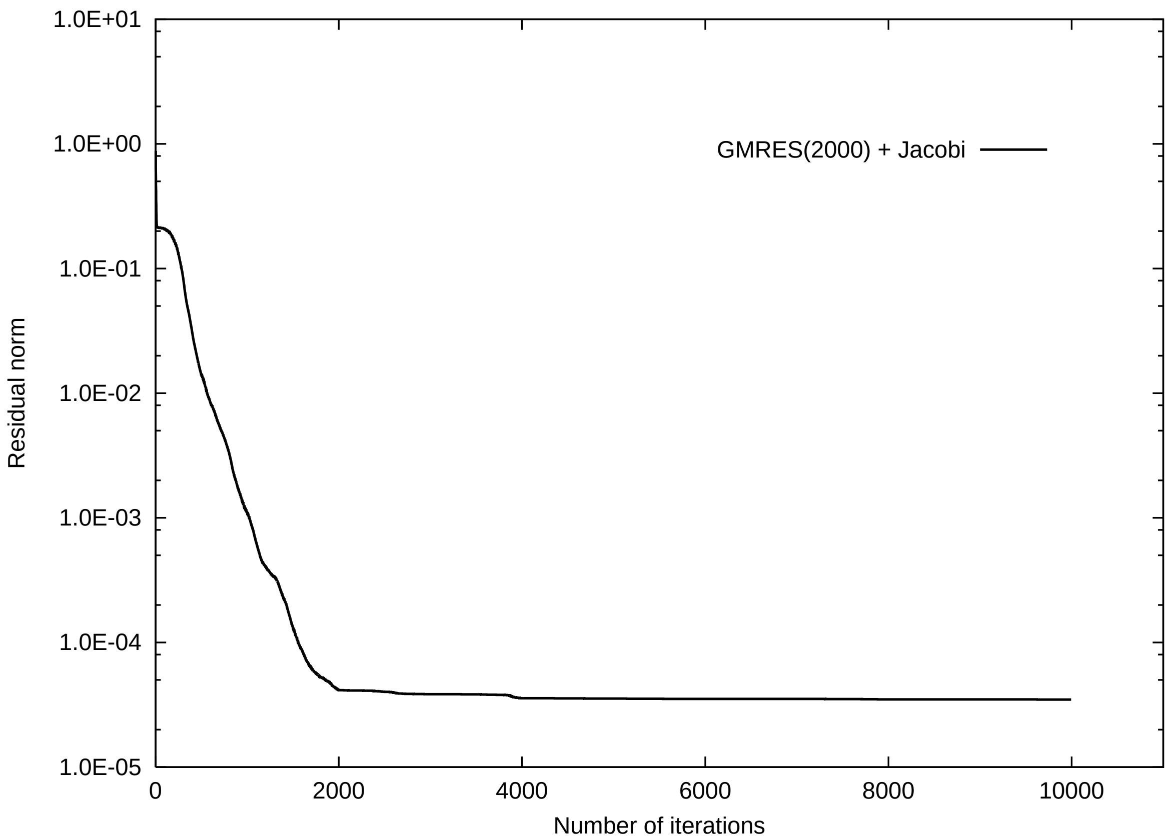

Figure 7: The profile of the residual norm $\|M^{-1}(b - Ax)\| / \|M^{-1}b\|$ versus the number of iterations.